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Engineering Notes

Passive Orbit Control for Space-Based Geo-Engineering

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I. Introduction

IN THIS Note, we consider using solar sail propulsion to stabilize a spacecraft about an artificial libration point. It has been demonstrated [1,2] that the constant acceleration from a solar sail can be used to generate artificial libration points in the Earth–sun three-body problem. This is achieved by directing the thrust due to the sail such that it adds to the centripetal and gravitational forces. These libration points have the potential for future space physics and Earth observation missions. Of particular interest is the possibility of placing solar reflectors at the L_1 artificial libration point to offset natural- and human-driven climate change [3,4]. One engineering challenge that presents itself is that these artificial libration points are highly unstable and require active control for station keeping. Previous work has shown that it is possible to stabilize a solar sail about artificial libration points using variations in both the pitch and yaw angles [2,5–7]. However, in a practical sense, solar sails are large structures and active control of the sail's attitude is a challenging engineering problem. Passive stabilization of such reflectors is to be investigated here to reduce the complexity of space-based geo-engineering schemes.

A possible alternative for stabilizing a sail about an artificial libration point is to fix the attitude of the solar sail and actuate the lightness number. The sail lightness number β is the ratio of the solar radiation pressure acceleration to the solar gravitational acceleration. This lightness number control, called β control, offers the possibility for passive stabilization of a sail about artificial libration points. β control has previously been used to stabilize artificial libration points by using it alongside a pitch angle control [5]; thus, active attitude control is still required. In this Note, we address the issue of using only β control to stabilize a solar sail about an artificial libration point. β control offers the possibility for passive control designs based on natural feedback laws such as $\partial\beta = -k\partial x$, where k is a constant parameter, $\partial\beta$ is the change in sail lightness number, and ∂x is the change in distance from the sun. One type of passive control that would naturally induce such a feedback could materialize through a conceptual "solar balloon." For example, as a balloon with a reflective surface is perturbed toward the sun and away from the artificial libration point ($\partial x < 0$), the balloon would expand due to the increase in temperature, thus increasing its surface area and, therefore, lightness number β ($\partial\beta > 0$). This increase in β would then accelerate the sail back toward the artificial libration point to ensure stability. Clearly, the converse would be true if the solar balloon is perturbed away from the sun. In addition, passive β control could be realized through the use of variable reflectance materials [8] or heat-sensitive actuators. We do not consider details of such mechanisms in this Note.

To assess the possibility of using β control, we use the setting of the circular restricted three-body problem (CRTBP) in which the massless body is a solar sail and the primaries are the sun and Earth, respectively. First, we determine the β controllability of the sail, that is, we compute the controllability matrix in the vicinity of the artificial libration point assuming only lightness number control. This illustrates that the sail is β controllable in the ecliptic plane, but not out of the plane. Second, we determine the β stability of the sail, that is, we compute the eigenvalues of the linearized closed-loop system to determine the linear stability properties of a β-controlled libration point. This illustrates that the libration point can be asymptotically stabilized in the ecliptic plane, but out of the plane, only the weaker form of Lyapunov stability at linear order (marginal stability) can be achieved, that is, the modes associated with the imaginary eigenvalues will have bounded amplitudes. In addition, a purely passive control $\partial\beta = -k\partial x$ is shown to be able to Lyapunov stabilize a sail about an artificial libration point.

Finally, the practical implications of these results are addressed through numerical simulation. Moreover, we show that, with large injection errors in the ecliptic plane, β control can be used to asymptotically stabilize the sail at an artificial libration point. Additionally, we show that with injection errors out of the plane, β control can be used to Lyapunov stabilize the solar sail about an artificial libration point in the full nonlinear model (in the sense that the sail trajectory is bound in the vicinity of the artificial libration point). In the case of the passive solar balloon, it is shown that Lyapunov stability can be achieved.

The classical CRTBP is modeled conveniently using a rotating frame [9] and is used as the basis for our solar sail model in the Earth–sun system. We consider a rotating coordinate system in which the primary masses are fixed on the x axis with the origin at the center of mass, the z axis is the axis of rotation, and the y axis completes the triad. We choose our units to set the gravitational constant, the sum of the primary masses, the distance between the primaries, and the magnitude of the angular velocity of the rotating frame to be unity. We shall denote by $\mu = 3 \times 10^{-6}$ the dimensionless mass of the smaller body m_2, the Earth; therefore, the mass of the larger body m_1, the sun, is given by $1 - \mu$. Denoting by $\boldsymbol{r}, \boldsymbol{r}_1$, and $\boldsymbol{r}_2$ the position of the sail with respect to the origin, m_1 and m_2, respectively, such that $\boldsymbol{r} = [x, y, z]^T$, $\boldsymbol{r}_1 = [x + \mu, y, z]^T$, and $\boldsymbol{r}_2 = [x + \mu - 1, y, z]^T$, the solar sail's equations of motion in the rotating frame are

$$\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2} + 2\boldsymbol{\omega} \times \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \boldsymbol{a} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) - \nabla V \equiv \boldsymbol{F} \tag{1}$$

with $\boldsymbol{\omega} = \hat{\boldsymbol{z}}$ and $V = -[(1-\mu)/r_1 + \mu/r_2]$, where $r_i = |\boldsymbol{r}_i|$ and $\boldsymbol{a} = [a_x, a_y, a_z]^T$. These differ from the classical equations of motion in the CRTBP by the radiation pressure acceleration term. In the case of the conceptual solar balloon, which for simplicity of exposition is assumed to be spherical, the radiation pressure acceleration will be in the direction of the sun–balloon line $\hat{\boldsymbol{r}}_1$; therefore,

$$\boldsymbol{a} = \beta \frac{(1-\mu)}{r_1^2} \hat{\boldsymbol{r}}_1 \tag{2}$$

β values up to 0.5 are considered and we plot all the possible artificial libration points for this β range. This is simply computed by setting $y = z = \dot{x} = \dot{y} = \dot{z} = 0$ in Eq. (1) and plotting β against the artificial libration point location $\boldsymbol{r}_e = [x_e, 0, 0]^T$, as shown in Fig. 1.

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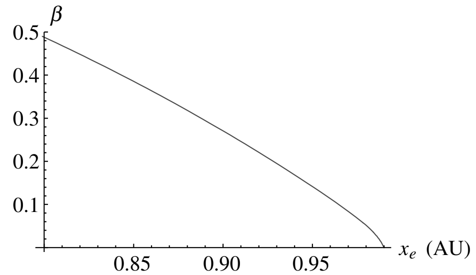


Fig. 1 Plot of the sail lightness number β against the artificial libration point in the Earth–sun system.

II. β Controllability

In this section, we determine the β controllability of the sail about an artificial libration point. First, we linearize about $\boldsymbol{r}_e$ by making the transformation $\boldsymbol{r} \to \boldsymbol{r}_e + \delta\boldsymbol{r}$, Taylor expanding $\boldsymbol{F}$ about $\boldsymbol{r}_e$, and neglecting the terms quadratic in $\delta\boldsymbol{r}$. Writing $\boldsymbol{X}(t) = (\delta\boldsymbol{r}, \delta\dot{\boldsymbol{r}})^T$, our linear system is

$$\dot{\boldsymbol{X}}(t) = A\boldsymbol{X}(t) \tag{3}$$

with

$$A = \begin{pmatrix} 0 & I \\ M & \Omega \end{pmatrix}, \qquad M = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}, \qquad \Omega = \begin{pmatrix} 0 & 2 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{4}$$

where $a = (\partial_x \boldsymbol{F}^x)|_e$, $b = (\partial_y \boldsymbol{F}^y)|_e$, and $c = (\partial_z \boldsymbol{F}^z)|_e$.

For a solar sail with lightness number control, we write Eq. (3) with a linear control term:

$$\dot{\boldsymbol{X}}(t) = A\boldsymbol{X}(t) + B\delta\beta \tag{5}$$

where $B = \begin{bmatrix} 0 & 0 & 0 & \frac{\partial a_x}{\partial \beta} & \frac{\partial a_y}{\partial \beta} & \frac{\partial a_z}{\partial \beta} \end{bmatrix}^T$.

The β controllability of this system about the artificial libration points are then determined by the rank of the controllability matrix $Q = [B \quad AB \quad A^2B \quad A^3B \quad A^4B \quad A^5B]$. As the rank of this 6×6 matrix is 4, the artificial libration points are not β controllable. However, if we set $z = 0$ in Eq. (1), it reduces to the planar solar sail CRTBP. Linearizing the system about the artificial libration point in the planar case reveals that Q is of full rank. Therefore, the libration points are β controllable in the ecliptic plane. In the proceeding section we illustrated that, despite the sail not being β controllable out of the plane, it is β stabilizable in the sense of Lyapunov about $\mathbf{r}_e$.

III. β Stabilizability

Setting $\delta\beta = 0$ in Eq. (5) yields the usual CRTBP linearized about the libration point L_1. In this case, the qualitative behavior in the vicinity of L_1 is representative of a saddle $\times$ center $\times$ center and is therefore unstable. However, in this section, we assess the β stability of the sail about the artificial libration points $\mathbf{r}_e$ by applying a linear feedback back control $\delta\beta = -K(\mathbf{X}(t))$ to Eq. (5) to yield the closed-loop system:

$$\dot{\mathbf{X}}(t) = (A - BK)\mathbf{X}(t) \tag{6}$$

If the real parts of the eigenvalues of the matrix $(A - BK)$ are all less than or equal to zero, then the libration points are Lyapunov stable in the sense that the modes associated with the imaginary eigenvalues will have bounded amplitudes at linear order; if their real parts are all less than zero, then they are asymptotically stable.

We initially take a near-term value of $\beta = 0.05$ and its corresponding artificial libration point $\mathbf{r}_e = [0.9804099, 0, 0]^T$. Following this, we define a simple gains matrix,

$$K = -[\rho_1 \quad 0 \quad 0 \quad \rho_2 \quad 0 \quad 0] \tag{7}$$

so that $\partial\beta = -\rho_1\partial x - \rho_2\partial\dot{x}$ and consider two cases: 1) $\rho = \rho_1 = \rho_2$, and 2) $\rho_1 \neq 0$, $\rho_2 = 0$. The case in which $\rho_2 = 0$ is for a purely passive control that reflects the feedback mechanism of a solar balloon.

Initially, we set $\rho_1 = \rho_2 = \rho$ and plot the corresponding real part of the eigenvalues of the matrix $(A - BK)$ for ρ up to 10 (see Fig. 2). We note that the imaginary parts of the eigenvalues remain qualitatively unchanged throughout and that the real parts of γ_5 and γ_6 remain zero for all time.

For $\rho = 0$, we have a center $\times$ saddle $\times$ center, as in the case of L_1 with no sail acceleration. There is an instantaneous bifurcation as ρ is increased above 0; for $0 < \rho < 4$, we have a stable spiral $\times$ saddle $\times$ center. An additional bifurcation occurs at approximately $\rho = 4$ and the qualitative local behavior about $\mathbf{r}_e$ for $\rho > 4$ is a stable spiral $\times$ stable node $\times$ center. Therefore, for values of $\rho > 4$, the artificial libration point $\mathbf{r}_e$ is β stabilizable in the sense of Lyapunov.

Second, we set $\rho_2 = 0$ and vary ρ_1 up to 10. In this case, the real parts of $\gamma_1, \gamma_2, \gamma_5$, and γ_6 are all zero. The real part of γ_3 is negative for $\rho_1 < 4$ and zero for $\rho_1 > 4$, and the real part of γ_4 is positive for $\rho_1 < 4$ and zero for $\rho_1 > 4$. Therefore, for $\rho_1 < 4$, it is not possible to β stabilize the artificial libration point. However, it is possible to Lyapunov stabilize the artificial libration point for $\rho_1 > 4$, where the closed-loop local behavior in the vicinity of the equilibrium point is a center $\times$ center $\times$ center. This suggests that a passive controller such as a solar balloon can Lyapunov stabilize itself about an artificial libration point at linear order.

We also consider incremental values of β up to 0.5 for both cases, $\rho = \rho_1 = \rho_2$ and $\rho_1 \neq 0$, $\rho_2 = 0$. For each case we find that the artificial libration point $\mathbf{r}_e$ is β stabilizable in the sense of Lyapunov.

To determine the linear stability of a closed-loop system of the form (6), it is also possible to use the Routh–Hurwitz stability criterion [10]; however, as some or all of the eigenvalues lie on the imaginary axis, this is not sufficient to prove nonlinear stability (see the Hartman–Grobman Theorem [11]). Therefore, we proceed to illustrate the nonlinear stability numerically by example. In the following section, we use simulation of a near-term sail ($\beta = 0.05$) at $\mathbf{r}_e$ to illustrate the nonlinear stability in the presence of injection errors.

IV. Practical Implications of the Results

If we place a solar sail with a near-term lightness number of $\beta = 0.05$ at its corresponding artificial libration point $\mathbf{r}_e = [0.9804099, 0, 0]^T$, it will diverge rapidly due to its inherent instability. However, using the gains matrix (7) with 1) $\rho_1 = \rho_2 = 10$ and 2) $\rho_1 = 10$ and $\rho_2 = 0$, small variations in β will maintain the sail in this position. In this section, we illustrate that with large injection errors of approximately 20,000 km in the ecliptic plane and velocity errors in the x–y directions of 200 ms^{-1} it is possible to asymptotically stabilize the sail onto the artificial libration point with $\rho_1 = \rho_2 = 10$. Figure 3a shows the sail's trajectory converging to $\mathbf{r}_e$,

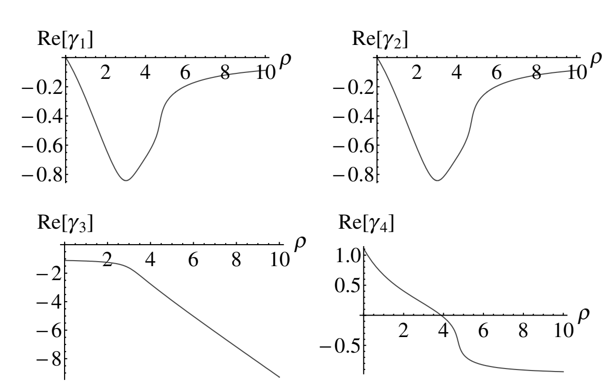
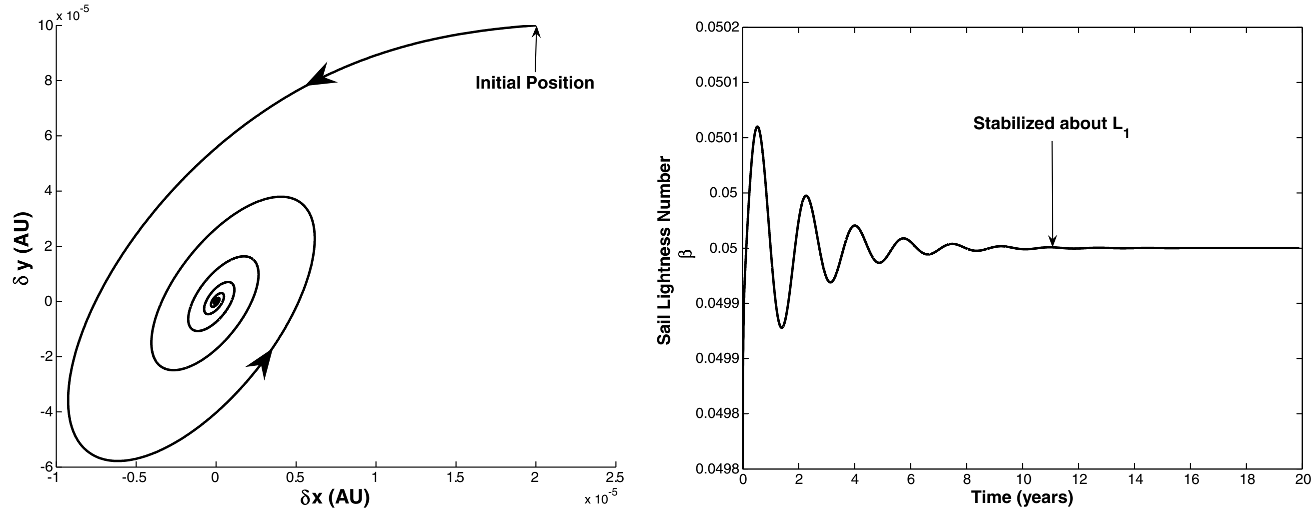
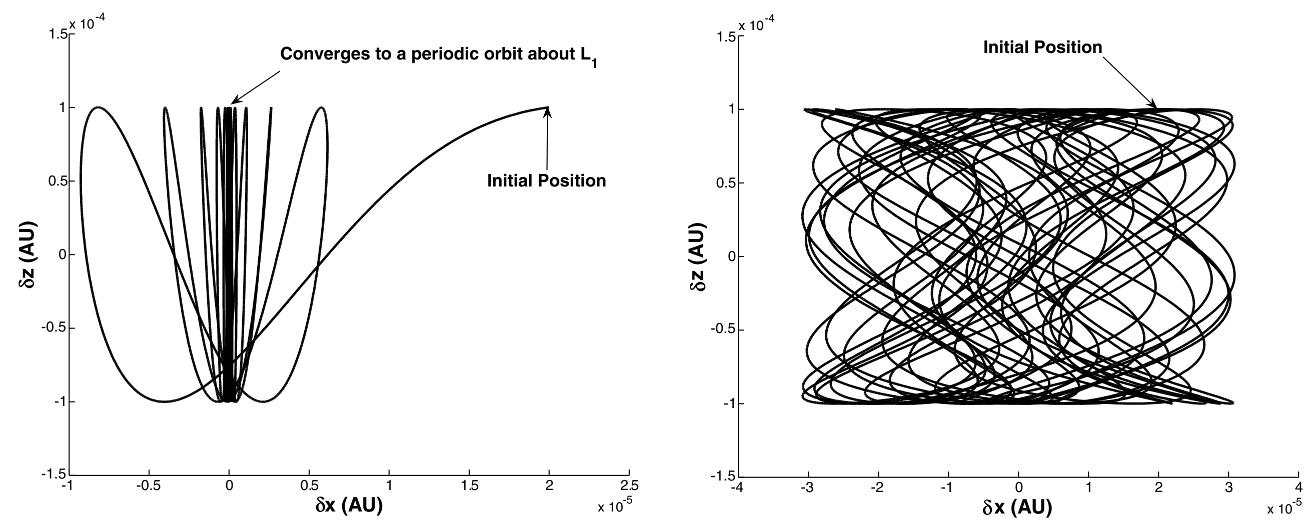


Fig. 2 Real parts of the eigenvalues with increasing control magnitude ρ.



a) b)

Fig. 3 β control asymptotically stabilizes the sail: a) sail asymptotically stabilizes about L_1 using β control, and b) β control variations required to drive onto and stabilize the sail about L_1.



a) b)

Fig. 4 β control Lyapunov stabilizes the sail: a) for $\rho_1 = \rho_2 = \rho = 10$, the sail converges to a periodic orbit about L_1 where the initial injection error in the z direction is equal to the amplitude of oscillation, and 2) for $\rho_1 = 10$, $\rho_2 = 0$, the sail is Lyapunov stabilized about the libration point.

where δx and δy are the distance from the artificial libration point, with the required variations in β illustrated in Fig. 3b.

Initially, as there is no injection error out of the plane, the β control can asymptotically stabilize the sail with $\rho_1 = \rho_2 = 10$. However, the sail is not β controllable out of the plane and implementing the same injection errors as before with a position error of 15,000 km in the z direction yields the variations in the x–z plane, as illustrated in Fig. 4a, in which δx and δz are the distance from the artificial libration point. In the case of the purely passive control with $\rho_1 = 10$, $\rho_2 = 0$, asymptotic stability is not possible in any direction, but it does stabilize about the artificial libration point in the sense of Lyapunov (the trajectory is bounded), as shown in Fig. 4b.

In Fig. 4a, the β control asymptotically stabilizes the sail in the ecliptic plane as in Fig. 3a, but oscillates in the z direction about $\mathbf{r}_e$ with an amplitude equal to the initial position error in the z direction (defined as ϵ). Therefore, the sail can only be β stabilized in the sense of Lyapunov ($x \to x_e, y \to 0, z \leq \epsilon$). For geo-engineering purposes, the initial z error must be less than the radius of the solar disk. In the case of complete passive control required for the implementation of a solar balloon, asymptotic stability is not possible in any direction but Lyapunov stability is. Therefore, the effectiveness of a purely passive β control is strongly dependent on the initial injection errors.

V. Conclusions

This Note has illustrated that a solar sail may stabilize its motion in the sense of Lyapunov about an artificial libration point using only small variations in its lightness number β. It has also been shown that the sail is β stabilizable in the sense of Lyapunov about the artificial equilibrium point despite the fact it is only β controllable in the plane. The implications of this are illustrated in a simulation, and we show that in the presence of injection errors in the ecliptic plane it is possible to asymptotically stabilize the sail. However, if there are injection errors in the z direction, the results illustrate that it is only possible to Lyapunov stabilize the sail about the artificial libration point. In addition, it is illustrated that, in the purely passive case, for example using a solar balloon, it is possible to achieve only Lyapunov stability if any injection error occurs.

Acknowledgments

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